Question 3 LO 4.3

$S = t^3 - 9t^2 + 15t + 2$

a Velocity $\frac{ds}{dt} = 3t^2 - 18t + 15$

$V = 3t^2 - 18t + 15$

b Velocity after 6 second

$V = 3(6)^2 - 18(6) + 15$

~~V = 72~~ $V = 108 - 108 + 15$

$V = 15 \text{ m/s}$

c $a = \frac{dv}{dt}(3t^2 - 18t + 15)$

$a = 6t - 18$

after 4 second

$a = 6(4) - 18$

$a = 24 - 18$ $\boxed{a = 6 \text{ m/s}^2}$

d $6t - 18 = 0$ $6t = 18$

$\frac{\cancel{6}t}{\cancel{6}} = \frac{18}{6}$ $t = 3$ second

$= 3s.$

Question 4 LO 4.4

1 $Y = 5x^3 + \cos 4x - \sin 3x$

$\frac{dy}{dx}\left(5x^3 + \cos(4x) - \sin(3x)\right)$

$= \frac{d}{dx}(5x^3) + \frac{d}{dx}(\cos(4x)) - \frac{d}{dx}(\sin(3x))$

$\frac{d}{dx}(5x^3) = 15x^2$, $\frac{d}{dx}(\cos(4x)) = -\sin 16x$

$\frac{d}{dx}(\sin 3x) = \cos 9x$

$= 15x^2 - \sin 16x - \cos 9x$

ii $Y = e^x - \sin 3x + 2x - 5$

$\frac{dy}{dx} e^x = e^x$ $\quad$ $\frac{d}{dx}(\sin(3x)) = \cos(3x) \cdot 3$

$\frac{d}{dx}(2x) = 2$ $\quad$ $\frac{d}{dx}(5) = 0$

$= e^x - 3\cos(3x) + 2$

iii $Y = e^{2x} + 5x^{-2}$

$\frac{d}{dx} e^{2x} = 2e^{2x}$

$\frac{d}{dx}(5x^{-2}) = -\frac{10}{x^3}$

$= 2e^{2x} - \frac{10}{x^3}$

Quetion 5 LO 4.5

i $y = (2x-4)(x^2-5)$

Apply the product rule

$$\frac{dy}{dx}(2x-4)(x^2-5) + \frac{dy}{dx}(x^2-5)(2x-4)$$

$$d/dx\,(2x-4) = 2$$

$$d/dx\,(x^2-5) = 2x$$

$$= 2(x^2-5) + 2x(2x-4)$$

$$= 6x^2 - 8x - 10$$

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ii $y = \frac{x^2+5}{x+7}$ Apply Quotient rule.

$$\frac{\frac{d}{dx}(x^2+5)(x+7) - \frac{d}{dx}(x+7)(x^2+5)}{(x+7)^2}$$

$$\frac{d}{dx}(x^2+5) = 2x$$

$$\frac{d}{dx}(x+7) = 1$$

$$\frac{2x(x+7) - 1(x^2+5)}{(x+7)^2}$$

$$= \frac{x^2+14x-5}{(x+7)^2}$$

Section 2 - Integral calculus

Question 1 LO 4.6

i $\int 16dx$

$\int 16dx \quad = \int 16dx = 16x + C$

$= 16x + C$

ii $\int 9x^3 + 7\cos(x)\, dx$

$= \int 9x^3\, dx + \int 7\cos(x)\, dx$

$\int 9x^3\, dx = \frac{9x^4}{4}$

$\int 7\cos(x)\, dx = 7\sin(x)$

$= \frac{9x^4}{4} + 7\sin x$

iii $\int e^{5x}\, dx \quad$ let $u = 5x$

$= \int e^u \frac{1}{5} du \quad = \frac{1}{5} e^u$

$= \frac{1}{5}\int e^u \cdot du \quad$ substitute back $u = 5x$

$= \frac{1}{5} e^{5x} + C$

Question 2 LO 4.7

i $A = \int_{-2}^{2} 5x^2 \, dx$

$5 \int_{-2}^{2} x^2 \, dx$

$5 \left[\frac{x^{2+1}}{2+1} \right]_{-2}^{2} = 5 \left[\frac{x^3}{3} \right]_{-2}^{2}$

$A = 5 \times \frac{16}{3} = 80/3$

---

ii $A = \int_{0}^{4} (2x^4 + 6x) \, dx$ Sum rule

$\int_0^4 2x^4 dx + \int_0^4 6x \, dx$

$\int_0^4 2x^4 dx = \frac{2048}{5}$, $\int_0^4 6x \, dx = 48$

$\frac{2048}{5} + 48 = \frac{2288}{5}$ $A = 457.6$

---

iii $A = \int_{\pi/2}^{\pi} \sin x \, dx = \left[-\cos(x) \right]_{\pi/2}^{\pi}$

$A = 1$

question 3 10.4.8

1 $y = 3x^2 + 6x - 5$

$f'(x) = 6x + 6$

$f'(x) > 0 : x > -1$

$f'(x) < 0 : x < -1$

plug $x = -1$ into $3x^2 + 6x - 5 = -8$

Minimun $(-1, -8)$

It has local Minimum $(-1, -8)$

---

2 $y = x^3 - 3x^2 - 9x + 7$

$f'(x) = 3x^2 - 6x - 9$

Critical points $x = -1$, $x = 3$

$f'(x) > 0$ $x < -1$ or $x > 3$

plug $x = -1$ into $x^3 - 3x^2 - 9x + 7 = 12$

Maximum $(-1, 12)$

plug $x = 3$ into $x^3 - 3x^2 - 9x + 7 = -20$

Minimum $(3 - 20)$

Maximum $(-1, 12)$, Minimum $(3, -20)$

Question 4 LO 4.9

i $V = \int_0^6 x^2 dx$

$= \left[\frac{x^{2+1}}{2+1}\right]_0^6$

$\left[\frac{x^3}{3}\right]_0^6$

$= 72$

ii $\int_0^1 \sin\pi x \, dx$

$\sin(\pi) x = 0$

$= \left[0 \cdot x\right]_0^1$

$\left[0\right]_0^1 = 0$

$= 0$

Quetion 5

iii $y = \frac{4x^2+8x}{3x+6}$ use quotient rule.

$\frac{dy}{dx}\left(\frac{4x^2+8x}{3x+6}\right)$

$\frac{\frac{d}{dx}(4x^2+8x)(3x-6) - \frac{d}{dx}(3x+6)(4x^2+8x)}{(3x+6)^2}$

$\frac{d}{dx}(4x^2+8x) = 8x+8$, $\frac{d}{dx}(3x+6) = 3$

$= \frac{(8x+8)(3x+6) - 3(4x^2+8x)}{(3x+6)^2} = \frac{4}{3}$

$= 4/3$

iv $y = (5x^2-7)^{12}$ use chain rule.

$12(5x^2-7)^{11} \frac{d}{dx}(5x^2-7)$

$\frac{d}{dx}(5x^2-7) = 10x$

$= 12(5x^2-7)^{11} \cdot 10x$

$= 120x(5x^2-7)^{11}$